

# Tensor gauge field localization in branes

M. O. Tahim,<sup>a,b</sup> W. T. Cruz,<sup>a,c</sup> and C. A. S. Almeida<sup>a</sup>

<sup>a</sup>*Departamento de Física - Universidade Federal do Ceará  
C.P. 6030, 60455-760 Fortaleza-Ceará-Brazil*

<sup>b</sup>*Departamento de Ciências da Natureza, Faculdade de Ciências,  
Educação e Letras do Sertão Central (FECLESC),*

*Universidade Estadual do Ceará, 63900-000 Quixadá-Ceará-Brazil and*

<sup>c</sup>*Centro Federal de Educação Tecnológica do Ceará (CEFETCE),*

*Unidade Descentralizada de Juazeiro do Norte, 63040-000 Juazeiro do Norte-Ceará-Brazil*

In this work we study localization of a Kalb-Ramond tensorial gauge field on a membrane described by real scalar fields. The membrane is embedded in an AdS-type five dimensional bulk space, which mimic a Randall-Sundrum scenario. First, we consider a membrane described by only a single real scalar field. In that scenarios we find that there is no localized tensorial zero mode. When we take into account branes described by two real scalar fields with internal structures, we obtain again a non-localized zero mode for a Kalb-Ramond tensorial gauge field. After modifying our model of one single scalar field by coupling the dilaton to the Kalb-Ramond field, we find that this result is changed. Furthermore, we analyze Kaluza-Klein massive modes and resonance structures.

PACS numbers: 11.27.+d, 11.15.-q, 11.10.Kk, 04.50.-h

Keywords: Field theories in higher dimensions, Kalb-Ramond field, Randall-Sundrum scenario

## I. INTRODUCTION

In scenarios containing extra dimensions in the background of membranes a very important subject is related to mechanisms of localization of several fields with different spins. Such importance is due to a very simple motivation: based on this idea all of the characteristics of a lower dimensional effective model must be obtained. Characteristics such that coupling constants and the masses of the fundamental particles depends on the size of the extra dimension and this fact may give new insights in order to understand some problems presented by the Standard Model.

There are at least two different scenarios of extra dimensions. The first one consider small and compact dimensions whose effects in lower energies is not observable. The second one regards the extra dimensions as having infinite size and they are not observable simply because we have not permission (an energy permission) to "walk" along them. The idea is that we live inside a membrane embedded in a spacetime of dimensionality bigger than 4. It is interesting to compare the differences between these two scenarios in models of field localization. For example, related to gauge vector fields of the Standard Model, it is well known that in a scenario where the extra dimension is compact the localization of this kind of field is not favored due to phenomenological constraints imposed by the Standard Model [1]. In the case where the extra dimension is infinite there is not localization unless the gauge field couples to the dilaton field [2]. We should try to understand the behavior of other sort of gauge fields in the context of membranes, in particular those with higher spins. The question we ask here is: can these fields have observable effects? Regarding this question, there is a new interest in theories containing gauge fields with higher spins. The basic reason for that is its existence on anti-de Sitter spaces [3]. Such characteristic signals for their relevance for AdS/CFT correspondence [4]. String theory also gives additional support to higher spin fields. Indeed, string theory contains an infinite number of higher spin fields with consistent interactions. In the low-tension limit their masses disappear. Massless higher spin theories are thus the natural candidates for the description for the low-tension limit of string theory at the semi-classical level. The hope is that the understanding of the dynamics of higher spin fields could help towards a deeper insight of string theory, which now is mainly based on its low-spin excitations and their low-energy interactions.

The main subject of this work is related to the Kalb-Ramond gauge field, a rank two antisymmetric field, which is the simplest case in the list of higher rank fields to be studied. This field appears in effective theories of specific low energy superstring models, as cited above, and can describe axion physics or torsion of a Riemannian manifold. When interpreted as torsion, it is known that this field, when studied in the Randall-Sundrum background, possesses a localized zero mode extremely suppressed by the size of the extra dimension [7]. Such a result leads the authors of this reference to speculate about a spacetime torsion in our universe, even a small one. On the other hand, in string theories the appearance of axions from the antisymmetric tensor fields is quite natural [9].

Specifically in this work we make an analysis of mechanisms of localization for the antisymmetric tensor gauge field in a scenario where the extra dimension has infinite size. The method we follow is quite different from that described in [7] since we study smooth AdS-like backgrounds. In order to impose such a condition it is necessary to implement the membranes of the model in a more realistic way. For such, we use kink defects embedded in a higher dimensional spacetime. In this way, we can avoid problems related to spacetime singularities presented in the Randall-Sundrum

idea.

We also make use of models containing several scalar fields whose solutions describe thin membranes and membranes with internal structures [10], including a scenario of the coupling of Kalb-Ramond and dilaton fields. In particular, in this last scenario, we present results about zero and massive modes of the Kalb-Ramond tensor field.

The organization of this work is as follows: in the second section solutions describing the gravitational background due to smooth membranes are studied; in the following section the Kalb-Ramond field is included in this scenario; in the fourth and fifth sections we introduce the dynamics of models containing two fields, including the dilaton; in the sixth section, the coupling between the dilaton and the Kalb-Ramond field is studied and finally, the last section is reserved to discussions about Kaluza-Klein massive modes.

## II. THE KINK AS A MEMBRANE

In this section we study the solutions for the Einstein's equations in the background of a single thin membrane. The membrane is a kink embedded in a  $D = 4 + 1$  spacetime. We study a spacetime background solution preserving four dimensional Lorentz symmetry. These solutions in general describe AdS-like spacetimes. The action for the model is [2]:

$$S = \int d^5x \sqrt{-G} [2M^3 R - \frac{1}{2}(\partial\phi)^2 - V(\phi)]. \quad (1)$$

In the action above the field  $\phi$  generates the membrane of the model,  $M$  is the Planck constant in  $D = 5$  and  $R$  is the curvature tensor. The equation of motion for the field  $\phi$  supports a kink solution even in a gravitational background. For this case, the ansatz for the spacetime metric is:

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (2)$$

Despite the warp factor  $A(y)$ , the metric above preserves  $D = 4$  Lorentz invariance. As usually, in this work a capital index like  $M$  stands for 0, 1, 2, 3, 4 while Greek letters stands for 0, 1, 2, 3. In order to make the model simpler we assume the field  $\phi$  and the function  $A$  as only dependent of the extra dimension  $y$ . The resultant equations of motion coming from the action (1) are [2]

$$R_{MN} - \frac{1}{2}G_{MN}R = \frac{1}{4M^3} \{ \partial_M \phi \partial_N \phi - G_{MN} [\frac{1}{2}(\partial\phi)^2 + V(\phi)] \}, \quad (3)$$

$$\frac{1}{\sqrt{-G}} \partial_M \{ \sqrt{-G} G^{MN} \partial_N \phi \} = \frac{\partial V}{\partial \phi}$$

For a situation without a gravitational background it is easy to show that, for the potential function  $V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$ , the kink modeling the membrane is given by  $\phi(y) = v \tanh(ay)$  where  $a^2 = \frac{\lambda v^2}{2}$ . This is a solution for the equation of motion for the scalar field. The equations of motion for the case of the AdS-like gravitational background described above are given by:

$$\frac{1}{2}(\phi')^2 - V(\phi) = 24M^3(A')^2, \quad (4)$$

and

$$\frac{1}{2}(\phi')^2 + V(\phi) = -12M^3 A'' - 24M^3(A')^2. \quad (5)$$

Note that the prime means derivative in respect to the extra dimension. By adding the two equations of motion above and integrating the result two times we easily see that, for the chosen kink solution, the function  $A(y)$  must be

$$A(y) = \frac{v^2}{72M^3} [4 \ln \cosh(ay) - \tanh^2(ay)] \quad (6)$$

where we have used  $A(0) = 0$  and  $A'(0) = 0$ . Note that the exponential factor constructed with this function is localized around the membrane and for large  $y$  it looks like the Randall-Sundrum solution [11]. An important characteristic of this solution is that for small perturbations of the metric it is possible to show that there is a non-massive gravitational mode trapped to the membrane [2]. The solution found here reproduces the Randall-Sundrum

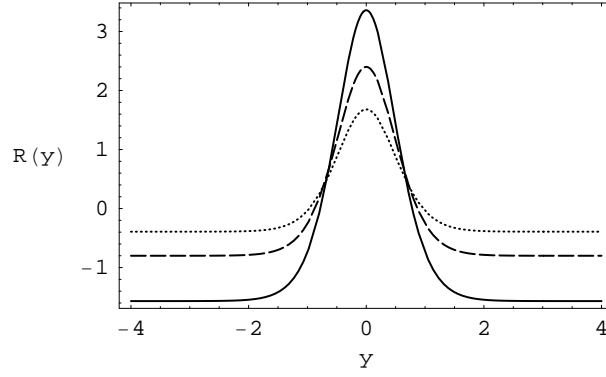


FIG. 1: Plots of the curvature invariant  $R(y)$  with  $\beta = 0.07$  (dotted line),  $0.1$  (dashed line),  $0.14$  (solid line) and  $a = 1$ .

model in an specific limit. The spacetime now has no singularity because we get a smooth warp factor (because of this, the model is more realistic). In fact this can be seen by calculating the curvature invariants for this geometry. For example, we obtain

$$R = -[8A'' + 20(A')^2], \quad (7)$$

where

$$A'(y) = -a\beta \tanh(ay) \left(2 + \frac{1}{\cosh^2(ay)}\right), \quad (8)$$

$$A''(y) = -\frac{3a^2\beta}{\cosh^4(ay)}, \quad (9)$$

and  $\beta = \frac{v^2}{36M^3}$ . Note that the Ricci scalar is finite and its behavior can be observed in Figure (1).

The potential  $V(\phi)$  must satisfy the equations of motion (4) e (5). Solutions for  $\phi(y)$  and  $A(y)$  will be

$$V(\phi) = -6M^3[A'' + 4(A')^2] \quad (10)$$

and its easy to see that this potential gets modified in order to support the solution proposed. Substituting the expression for  $A(y)$  and expressing the potential in terms of the scalar field we arrive at

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 - \frac{\lambda}{108M^3}\phi^2(\phi^2 - 3v^2)^2. \quad (11)$$

In the limit where  $M \rightarrow \infty$  the potential coincides with the standard double well potential.

### III. THE KALB-RAMOND FIELD

In this section we study the behavior of the Kalb-Ramond field in the gravitational background described in the last section. The main subject here is to try to detect a massless zero mode of the Kalb-Ramond field localized on the membrane of the model described here. Note that the extra dimension has infinite size, an important detail in order to achieve the final conclusion of this work. The action for the model is given by

$$S = \int d^5x \sqrt{-G} [2M^3 R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - H_{MNL}H^{MNL}] \quad (12)$$

where  $H_{MNL} = \partial_{[M}B_{NL]}$  is the field strength for the Kalb-Ramond field. The equation of motion for the field  $B_{MN}$  is easily obtained.

First, we obtain the following equation:

$$\partial_Q(\sqrt{-G}H_{MNL}G^{MQ}G^{NR}G^{LS}) = 0. \quad (13)$$

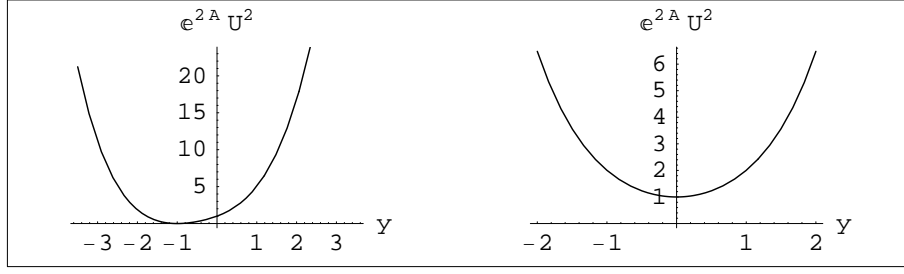


FIG. 2: Plots of  $U(y)^2 e^{2A(y)}$  with  $A(y)$  given by eq.(6) for  $U(y) = y + 1$  (left),  $U(y) = 1$  (right),  $\beta = 0.5$  and  $a = 1$ .

Now, we consider a gauge choice as  $B_{\alpha 5} = \partial_\mu B^{\mu\nu} = 0$ . Working with the part dependent on the extra dimension, we can rewrite the equation of motion above as

$$e^{-2A} \partial_\mu H^{\mu\gamma\theta} - \partial_y H^{y\gamma\theta} = 0. \quad (14)$$

Here we can split the variables through the following ansatz:

$$B^{\mu\nu}(x^\alpha, y) = b^{\mu\nu}(x^\alpha)U(y) = b^{\mu\nu}(0)e^{ip_\alpha x^\alpha}U(y), \quad (15)$$

where  $p^2 = -m^2$ . Therefore, we can rewrite  $H^{MNL}$  as  $h^{\mu\nu\lambda}U(y)$ , and the equation of motion becomes

$$\partial_\mu h^{\mu\nu\lambda}U(y) - e^{2A} \frac{d^2 U(y)}{dy^2} b^{\nu\lambda} e^{ip_\alpha x^\alpha} = 0. \quad (16)$$

The function  $U(y)$  carries all the information about the extra dimension and obeys the following equation:

$$\frac{d^2 U(y)}{dy^2} = -m^2 e^{-2A(y)} U(y). \quad (17)$$

For the case where  $m^2 = 0$  its solution is quite simple:  $U(y) = cy + d$ , where  $c$  and  $d$  are constants. Another solution is  $U(y) \equiv cte$ .

Now it is important to analyze the effective action in  $D = 4$  for the Kalb-Ramond field. The way chosen for this is based in the procedure of dimensional reduction. We focus our attention specifically on the massless zero mode that appears from that reduction. For the metric described in the previous section we have the following:

$$S \sim \int \sqrt{-G} d^5 x (H_{MNL} H^{MNL}) = \int dy U(y)^2 e^{-2A(y)} \int d^4 x (h_{\mu\nu\alpha} h^{\mu\nu\alpha}). \quad (18)$$

Given the solution (6) for the warp factor  $A(y)$ , we can see that for both type of solutions  $U(y)$  obtained above, the integral in the  $y$  variable in the effective action for the Kalb-Ramond zero mode is not finite. This can be easily seen if we analyse the plot of the function of  $y$  which is supposed to be integrated. Indeed, for the solution  $U(y) = cy + d$ , and for the solution  $U(y) \equiv cte$  as showed in Fig. (2), the integral in  $y$  do not converge.

Therefore, there is not localization of the Kalb-Ramond field on the membrane in the conditions described. This result should not be a surprise: the gauge vector field suffers the same problem.

#### IV. MEMBRANE WITH INTERNAL STRUCTURES

Due to the result of the last section, we may ask if there is some way to modify our model in order to change the final conclusion. Models containing more fields may, in principle, give us different results because of its natural and rich amount of information. Here we choose models that presents membranes with internal structures. Membranes with internal structures can be described by models with two real scalar fields. Indeed, in Ref. [12], Bazeia *et. al.* introduced a model which supports Bloch walls, which in turn, exist in ferromagnetic systems [14], and already have internal structures. Following ideas where domain walls are used in order to generate branes in scenarios where scalar fields couple with gravity, Bazeia and Gomes proposed the so called Bloch Brane [10]. This brane model can be thought as an alternative to the infinitely thin brane model, which, as pointed out in [15], may be a very artificial construction.

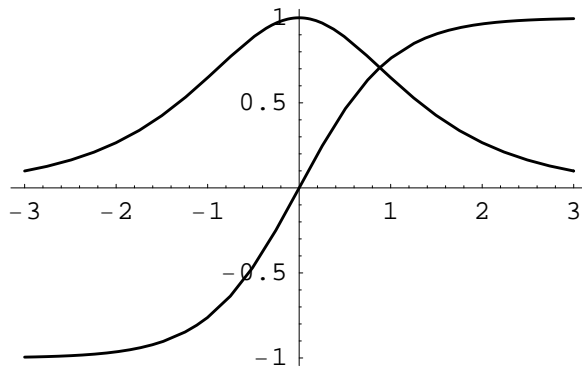


FIG. 3: The kink-like object represents the membrane while the lump-like object represents the internal structure.

To the best of our knowledge, there are no previous study about localization of fields in that type of brane. It is worthwhile to mention, however, a recent work by Gomes [16], which study **gravity** localization in Bloch Branes.

We now study a model of two real scalar fields. The spacetime has the same characteristics as those discussed in the second section:  $ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$ . The question is the same: can this model trap to a membrane a zero-mode of the Kalb-Ramond field? The difference here is that the membrane has internal structures as explained above: we are discussing now a Bloch-type brane. We can see in Fig. (3) the profile of the solution for a model of two real scalar fields.

We get a membrane which have an internal structure due to the scalar field denoted by  $\chi$ . This structure gives rise to other defects (which are called domain ribbons) inside the membrane generated by the field  $\phi$  [12]. We must verify, therefore, if the existence of this internal structure may give us additional information in order to localize the Kalb-Ramond gauge field.

The model is basically given by the following action:

$$S = \int d^4x dy \sqrt{-G} \left[ \frac{1}{4}R + \frac{1}{2}\partial_a\phi\partial^a\phi + \frac{1}{2}\partial_a\chi\partial^a\chi - V(\phi, \chi) \right], \quad (19)$$

where the metric is

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2. \quad (20)$$

The solutions we need to simulate the brane with internal structure are given by:

$$\phi(x) = \tanh(2rx) \quad (21)$$

$$\chi(x) = \pm \sqrt{\frac{1}{r} - 2\text{sech}(2rx)} \quad (22)$$

Following the procedure outlined in the previous sections we can easily find the warp factor as given by ( $r$  is a parameter of the model)

$$\exp[2A(y)] = \cosh^{-4}(2ry) \exp\left[\frac{2}{9r}(1 - 3r) \tanh^2(2ry)\right]. \quad (23)$$

For the effective action in  $D = 4$  of the Kalb-Ramond field we should take the inverse of this factor. Now, for the function  $U(y)$  constant, as discussed in the last section, we reobtain the result of non-localization of a tensorial zero-mode. The Bloch brane does not change the setup for the Kalb-Ramond field.

## V. THE DILATON

In this section we continue to study models containing more than one scalar field. Due to the result of the last section, it is natural to ask if there is some coupling between the Kalb-Ramond field and another field in order to

provide the localization of a tensorial zero mode. In analogy with the work of Kehagias and Tamvakis [2], where it is shown that the coupling between the dilaton and a vector gauge field produces localization of the latter, we introduce here the coupling between the dilaton and the Kalb-Ramond field. The coupling is motivated in low energy superstring theories [17].

Nevertheless, before analyzing the coupling, it is necessary to obtain a solution of the equations of motion for the gravitational field in the background of the dilaton and the membrane. For such, we introduce the following action [2]:

$$S = \int d^5x \sqrt{-G} [2M^3 R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\pi)^2 - V(\phi, \pi)]. \quad (24)$$

Note again that we are working with a model containing two real scalar fields. The field  $\phi$  again plays the role of to generate the membrane of the model while the field  $\pi$  represents the dilaton. The potential function now depends on both scalar fields. Furthermore, the Ricci scalar depends on a special form of the dilaton field, as we shall see ahead. But, the key point here is that it is assumed a new ansatz for the spacetime metric:

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2B(y)} dy^2. \quad (25)$$

The equations of motion are given by

$$\frac{1}{2}(\phi')^2 + \frac{1}{2}(\pi')^2 - e^{2B(y)} V(\phi, \pi) = 24M^3(A')^2, \quad (26)$$

$$\frac{1}{2}(\phi')^2 + \frac{1}{2}(\pi')^2 + e^{2B(y)} V(\phi, \pi) = -12M^3 A'' - 24M^3(A')^2 + 12M^3 A' B', \quad (27)$$

$$\phi'' + (4A' - B')\phi' = \partial_\phi V, \quad (28)$$

and

$$\pi'' + (4A' - B')\pi' = \partial_\pi V. \quad (29)$$

In order to solve that system, we use a so-called superpotential function  $W(\phi)$ , defined by  $\phi' = \frac{\partial W}{\partial \phi}$ , following the approach of Kehagias and Tamvakis [2]. The particular solution regarded follows from choosing the potential  $V(\phi, \pi)$  and superpotential  $W(\phi)$  as

$$V = e^{\frac{\pi}{\sqrt{12M^3}}} \left\{ \frac{1}{2} \left( \frac{\partial W}{\partial \phi} \right)^2 - \frac{5}{32M^2} W(\phi)^2 \right\}, \quad (30)$$

and

$$W(\phi) = va\phi \left( 1 - \frac{\phi^2}{3v^2} \right). \quad (31)$$

Following the procedure it is easy to obtain first order differential equations whose solutions are solutions of the equations of motion (26-29) above, namely

$$\pi = -\sqrt{3M^3} A, \quad (32)$$

$$B = \frac{A}{4} = -\frac{\pi}{4\sqrt{3M^3}}, \quad (33)$$

and

$$A' = -\frac{W}{12M^3}. \quad (34)$$

The solutions for these new set of equations are the following:

$$\phi(y) = v \tanh(ay), \quad (35)$$

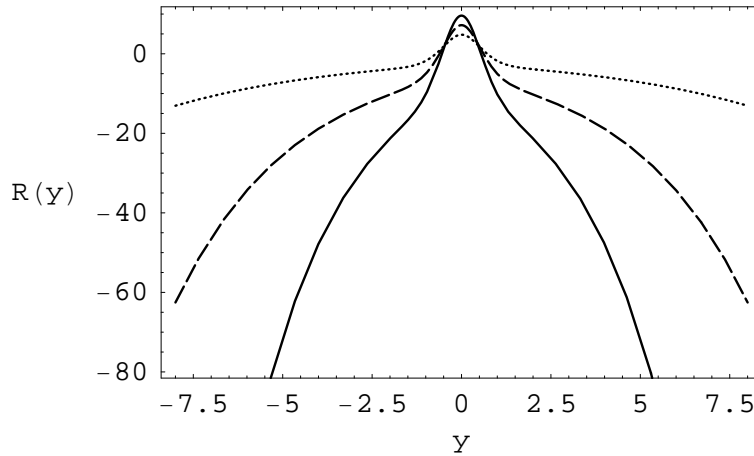


FIG. 4: Plots of the solution of the curvature invariant  $R(y)$  with  $\beta = 0.07$  (dotted line),  $0.1$  (dashed line),  $0.14$  (solid line),  $a = 1$  and  $3M^3 = 1$ .

$$A(y) = -\beta \ln \cosh(ay)^2 - \frac{\beta}{2} \tanh(ay)^2, \quad (36)$$

and

$$\pi(y) = \beta \sqrt{3M^3} (\ln \cosh(ay)^2 + \frac{1}{2} \tanh(ay)^2). \quad (37)$$

Following the argumentation in Ref. [2], it is possible to see that, by the linearization of the geometry described in this section, this model supports a massless zero mode of the gravitational field localized on the membrane, even in the dilaton background.

On the other hand the dilaton contribution makes the spacetime singular. However this kind of singularity is very common in D-brane solutions in string theory (the dilaton solution is divergent). The Ricci scalar for this new geometry is now given by

$$R = -[8A'' + 18(A')^2]e^{\frac{\pi}{2\sqrt{3M^3}}}, \quad (38)$$

where the dilaton has an important contribution. The new behavior of the Ricci scalar with the dilaton coupling can be observed in Figure (4).

What is interesting here is that this singularity disappears if we lift the metric solution to  $D = 6$ . In this case the dilaton represents the radius of the sixth dimension [2].

The next step is a review of the behavior of the Kalb-Ramond gauge field in this new background. Another important point is the analysis of stability of the model. The mixing of scalar parts of all fields of this model should be understood in order to see all aspects of localization. However it is subject for a next paper. In this aspect some authors have already made some developments [13].

It is worthwhile to point out that in scenarios where gravitational and scalar field fluctuations are considered, perturbations of the metric in transversal no trace sector splits from the scalar fields and then localized zero modes for the graviton are obtained. However, and we must emphasize this, despite the fact that in some models RS-type the Kalb-Ramond field could be included **with** the graviton, in our case the inclusion of the KR field is made through back reaction and therefore that field is **not** a constructive part of the geometry of the bulk.

## VI. REVIEWING THE KALB-RAMOND FIELD

The main subject of this section is to verify if the coupling between the dilaton and the tensor gauge field is able to produce localization of the latter. The dilaton coupling introduces the following modification in the action for the tensor gauge field [18, 19]:

$$S \sim \int \sqrt{-G} d^5x (e^{-\lambda\pi} H_{MNL} H^{MNL}). \quad (39)$$

Therefore, we must analyze the equations of motion of the tensor gauge field in the dilaton background. The new equation of motion is:

$$\partial_M(\sqrt{-G}G^{MP}G^{NQ}G^{LR}e^{-\lambda\pi}H_{PQR}) = 0. \quad (40)$$

With the gauge choice  $B_{\alpha 5} = \partial_\mu B^{\mu\nu} = 0$  and with the separation of variables  $B^{\mu\nu}(x^\alpha, y) = b^{\mu\nu}(x^\alpha)U(y) = b^{\mu\nu}(0)e^{ip_\alpha x^\alpha}U(y)$  where  $p^2 = -m^2$ , it is obtained the differential equation giving us information about the extra dimension, namely

$$\frac{d^2 U(y)}{dy^2} - (\lambda\pi'(y) + B'(y))\frac{dU(y)}{dy} = -m^2 e^{2(B(y)-A(y))}U(y). \quad (41)$$

For the zero mode,  $m = 0$ , a particular solution of the equation above is simply  $U(y) \equiv cte$ . This is enough for the following discussion. The effective action for the zero mode in  $D = 5$  is

$$S \sim \int \sqrt{-G}d^5x(e^{-\lambda\pi}H_{MNL}H^{MNL}) = \int dyU(y)^2 e^{(-2A(y)+B(y)-\lambda\pi(y))} \int d^4x(h_{\mu\nu\alpha}h^{\mu\nu\alpha}). \quad (42)$$

For  $U(y)$  as a constant and given the solutions for  $A(y)$ ,  $B(y)$  e  $\pi(y)$ , as in the last section, it is possible to show clearly that the integral in the  $y$  variable above is finite if  $\lambda > \frac{7}{4\sqrt{3M^3}}$ , providing the possibility of localization of the zero mode associated to the Kalb-Ramond field in the dilaton background.

## VII. KALUZA-KLEIN MASSIVE MODES

In this section we study the massive spectrum of the Kalb-Ramond tensor field. This analysis allow us to detect the presence of resonant mode solutions of the equation of motion dependent of the extra dimension. The dilaton coupling was of great importance to get a localized zero-mode, as we have seen in Section (6). Then we will follow our analysis of massive modes considering the same dynamics of dilaton coupling adopted in the previous section. For this, initially we must transform the equation of motion for the extra dimension (41), in an equation of the Schroedinger type.

Therefore we take the values of  $\pi(y)$  and  $B(y)$  to rewrite (41) in terms of  $A(y)$  and its derivatives on the following form,

$$\left\{ \frac{d^2}{dy^2} - \alpha A' \frac{d}{dy} \right\} U(y) = -m^2 e^{-\frac{3}{2}A} U(y), \quad (43)$$

were,

$$\alpha = \frac{1}{4} - \sqrt{3M^3}\lambda. \quad (44)$$

In order to get an equation of the Schroedinger type from the equation above we must proceed with the following mapping,

$$y \rightarrow z = f(y), \quad U = \Omega \bar{U}. \quad (45)$$

The conditions to get an equation of the Schroedinger type must indicate the form of the function  $\Omega$ . Therefore, we cannot have first derivative terms, and the right side of the required equation must contains the constant  $m^2$ . We will have then,

$$\Omega = e^{(\frac{\alpha}{2} + \frac{3}{8})A}, \quad \frac{dz}{dy} = e^{-\frac{3}{4}A}. \quad (46)$$

From the transformations (45), our Schroedinger-like equation can be written as,

$$\left\{ -\frac{d^2}{dz^2} + \bar{V}(z) \right\} \bar{U} = m^2 \bar{U}, \quad (47)$$

where the potential  $\bar{V}(z)$  assumes the form,

$$\bar{V}(z) = e^{\frac{3}{2}A} \left[ \left( \frac{\alpha^2}{4} - \frac{9}{64} \right) (A')^2 - \left( \frac{\alpha}{2} + \frac{3}{8} \right) A'' \right]. \quad (48)$$



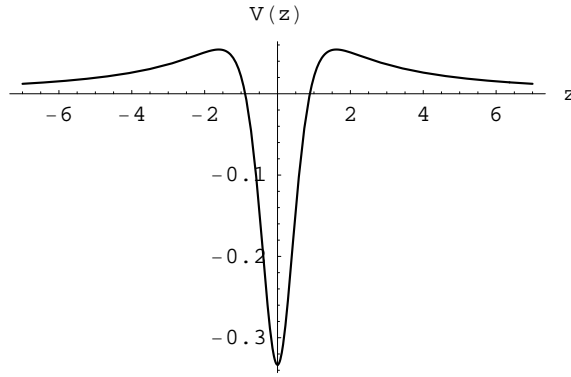


FIG. 5: Plots of potential  $\bar{V}(z)$  were we put  $\frac{v^2}{72M^3} = 1$

We can write the potential in function of the derivatives respect to  $z$ ,

$$\bar{V}(z) = \left[ \beta^2 (\dot{A})^2 - \beta \ddot{A} \right] \quad (49)$$

where,

$$\beta = \frac{\alpha}{2} + \frac{3}{8} \quad (50)$$

The equation (47) will only present finite solution if we consider  $\sqrt{3M^3}\lambda > 1$ , which it is compatible with the localization of zero-modes as described in the previous section.

We plot the form of the potential in the Figure (5), which suggests the possibility of resonant modes. As we can note in Eq. (48), the potential is such that  $\bar{V}(z) \rightarrow 0$  when  $z \rightarrow \infty$ . This excludes the possibility of gaps in the continuous spectrum.

It is interesting to point out that the Schroedinger-type equation (47) can be written in the supersymmetric quantum mechanics scenario as follows,

$$Q^\dagger Q \bar{U}(z) = \left\{ \frac{d}{dz} - \beta \dot{A} \right\} \left\{ \frac{d}{dz} + \beta \dot{A} \right\} \bar{U}(z) = -m^2 \bar{U}(z) \quad (51)$$

From the form of the Eq. (51), we exclude the possibility of normalized negative energy modes existence. On the other hand, we exclude also the possibility of the presence of tachyonic modes, which is a necessary condition to keep the stability of gravitational background.

We cannot find analytical solution of the massive modes wave function in Schroedinger equation. However we will be able to analyze the solution for  $\bar{U}$  by numerically solving the equation (47). We plot in Figure (6) the wave function so obtained for two values of  $m^2$ . As we can observe, the wave function oscillates quickly for a moderate value of  $m^2$  and reduces its period for small values of  $m^2$ . As mentioned in Ref. [10], this behavior of the wave function suggests a free motion in the bulk, but no imprisonment in the membrane.

In a close resemblance to the analysis of Refs. [20, 21], for some values of mass, our plane wave solutions can assume very high amplitudes inside the brane and this may be understood in terms of resonance structures. In figure (6) we analyze only two modes in which we observe only variations in the period of the oscillations without great values for  $\bar{U}(z)$  at  $z = 0$ . Therefore these two modes do not present resonances. However, as mentioned in Ref. [22], in order to detect the presence of resonances we must know the value of our solutions inside the membrane as function of the mass. In this way we will be searching a great number of values of mass capable to produce great amplitudes in  $z = 0$ .

The equation for massive modes written in the form of Eq.(51) allows us to interpret  $|\chi \bar{U}(z)|^2$  as a probability to find the mode in the position  $z$ , where  $\chi$  is a normalization constant. Therefore, we must calculate  $|\chi \bar{U}(0)|^2$  as a function of the mass in order to determine the intensity of the modes on the brane.

First, we solve numerically the Eq. (47) for a sequence of mass values in the interval  $0 < m < 1$  with a steep of  $10^{-3}$ . We have chosen this interval because the potential cause just a little perturbation for the modes for which  $m^2 \gg V_{max}$  [21]). Hence, in the case of existing a resonant structure, we hope to find it for  $0 < m < 0.24$ .

In order to normalize our plane wave function, we restrict the solutions of each mode to the region  $-100 < z < 100$  and we extract a normalization constant  $\chi$  for each correspondent value of mass. Then, we interpolate our data bank and we construct the function  $N_m = |\chi \bar{U}_0(m)|^2$  which give us the probability to find the modes in  $z = 0$  as function

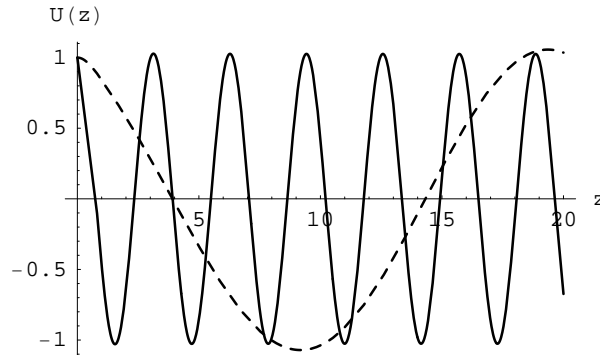


FIG. 6: Plots of wave function for  $m^2 = 4$  (solid line), and  $m^2 = 0.1$  (dashed line), where we put  $\frac{v^2}{72M^3} = 1$ .

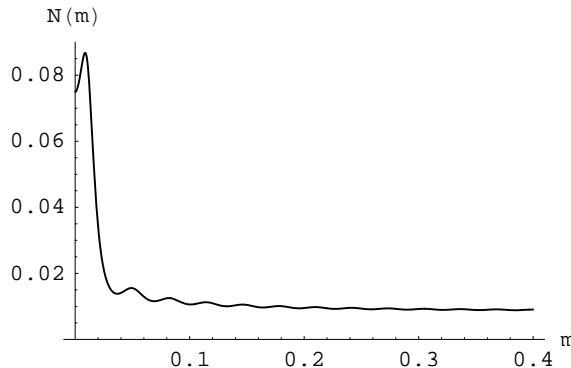


FIG. 7: Plots of  $N(m)$  with  $\sqrt{3M^3}\lambda = 2$

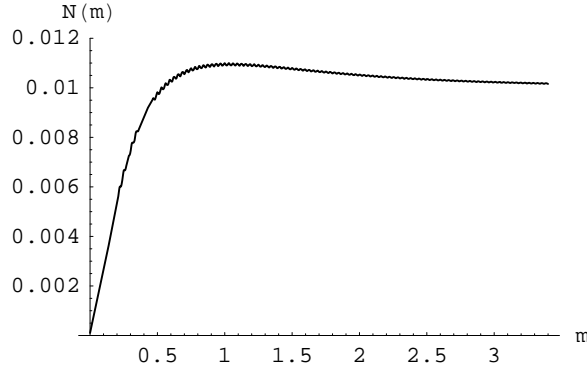
of  $m$ . We plot  $N(m)$  in the figure (7). As we can see, there is a resonance peak near  $m = 0$ , or more precisely, at  $m = 0.009$ . The analysis of the graphic (7) shows that for  $m = 0$  we have a big value for  $N(m)$ , assuring that we have a zero mode localization.

On the other hand, the coupling of the modes with matter in the brane is related to the normalized wave function amplitude in  $z = 0$  [21]. In our case, we impute that relation to the function  $N(m)$ . As we expected, non-massive modes show a high value for  $N(m)$ , considering that they are coupled to the matter in the brane. The resonance in  $m = 0.009$  shows us that only very light modes couple to the matter in a magnitude of the order of that of the zero modes. Furthermore, as we can see in Fig. (7), for heavy modes,  $N(m)$  tends to a constant value. Therefore, we can conclude that these modes have a weak coupling, if we compare them with the coupling of the very light ones.

An interesting additional point consists in testing the consistency of the function  $N(m)$  taking in to account the presence of the dilaton. In section III, when we consider the brane without internal structures, the zero mode of the KR field was not localized. Hence, if we consider that scenario, the function  $F(m)$  must be changed. In the examples discussed above, we have used  $\sqrt{3M^3}\lambda = 2$ . Therefore, in order to make the dilaton coupling disappears, we must to use  $\lambda = 0$  and recalculate the function  $N(m)$ . As we can note the zero mode coupling is highly suppressed compared to the massive modes and the resonance disappears. That result agreed with our results of the section III, namely, if we do not consider the dilaton coupling we obtain non localized modes again.

## VIII. CONCLUSIONS

In this work we have made a study of localization of zero and massive modes of a Kalb-Ramond gauge field on several types of branes. In a first case, we have just seen the contribution of the spacetime geometry in the behavior of a possible localized zero-mode. We have chosen an AdS-like geometry generated by a model of real scalar fields in five dimensions. We have shown that the solutions for this case does not guarantee the presence of zero modes trapped to the membrane. The reason for such a behavior is due to the infinite size of the extra dimension which makes the effective action for the Kalb-Ramond field in  $D = 4$  strongly divergent. Stepping further, we have added

FIG. 8: Plots of  $N(m)$  with  $\lambda = 0$ 

the dilaton field through a coupling to the Kalb-Ramond field in order to evaluate if this new ingredient produces normalized zero modes. The answer we have found is positive within certain conditions over the coupling constant  $\lambda$ , i.e, the dilaton contributes to localize such a zero mode, as in the case for the vector gauge field already discussed in Literature.

After this we address the issue of zero mode confinement in thick branes, in particular those called Bloch branes, which exhibit an internal structure, based in two real scalar fields and we have found again a negative answer for the localization of the Kalb-Ramond zero mode. Nevertheless, in the presence of the dilaton background we have zero mode localization. In a certain way, the results for the Kalb-Ramond gauge field without the dilaton background are corroborated by the analogous study in the Randall-Sundrum scenario where the extra dimension is compacted in an orbifold [7]. In that case there is a zero mode but strongly suppressed by the size of the extra dimension. Effectively, in the Randall-Sundrum scenario there is no dynamical Kalb-Ramond gauge field in  $D = 4$ . Finally, we study the Kaluza-Klein massive modes of Kalb-Ramond field. Using an approach of supersymmetric quantum mechanics in a Schrodinger-like equation, we exclude the possibility of gaps in the continuous spectrum.

By numerically solving the Schrodinger like equation of motion, initially for two modes, we get solutions of plane-waves, an indication of non localization. Knowing the variation of our solutions in the brane as function of mass, we detect the presence of a resonant modes. Therefore, from an analysis of the resonance structure we conclude that heavy massive modes have a weak coupling, if we compare they with the coupling of the very light ones. In addition, if we consider a situation where the coupling of the dilaton vanishes, the resonance disappears and the zero mode is highly suppressed compared to the massive modes.

The authors would like to thank Fundação Cearense de apoio ao Desenvolvimento Científico e Tecnológico (FUNCAP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) for financial support. Also, we are indebted to anonymous referees whose comments have helped us a lot to improve the manuscript.

- 
- [1] H. Davoudias, J.L. Hewett, T.G. Rizzo, Phys. Lett. B473, 43 (2000).
  - [2] A. Kehagias and K. Tamvakis, *Localized gravitons, gauge bosons and chiral fermions in smooth spaces generated by a bounce*, Phys. Lett. B504, 38 (2001).
  - [3] C. Fronsda, Phys. Rev. D20, 848 (1979); J. Fang and C. Fronsda, Phys. Rev. D22, 1361 (1980); V. E. Lopatin and M. A. Vasiliev, Mod. Phys. Lett. A 3, 257 (1988).
  - [4] C. Germani, A. Kehagias, Nucl. Phys. B725, (2005) 15-44.
  - [5] O. DeWolfe, D.Z. Freedman, S.S. Gubser, A. Karch, Phys. Rev. D62 046008(2000).
  - [6] D. Bazeia, A.R. Gomes, JHEP 0405, 012 (2004).
  - [7] B. Mukhopadhyaya, S. Sen, S. SenGupta, Phys. Rev. Lett. 89, 121101 (2002).
  - [8] B. Mukhopadhyaya, Siddhartha Sen, Somasri Sen, S. SenGupta, Phys. Rev. D70, 066009 (2004).
  - [9] E. Witten, *Some Properties Of  $O(32)$  Superstrings*, Phys. Lett. B 149, 351 (1984).
  - [10] D. Bazeia, A. R. Gomes, JHEP 0405, 012 (2004).
  - [11] L. Randall, R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); *ibid* 83, 4690 (1999).
  - [12] D. Bazeia, R. F. Ribeiro, M. M. Santos, Phys. Rev. D54, 1852 (1996).
  - [13] S. Kobayashi, K. Koyama, J. Soda, Phys. Rev. D65, 064014, (2002).
  - [14] D. Walgraef, *Spatio-temporal pattern formation*, Springer-Verlag, New York, 1997.
  - [15] F. Bonjour, C. Charmousis, and R. Gregory, Class. Quantum Grav. 16, 2427 (1999).
  - [16] A. R. Gomes, *Gravity on the Bloch Brane*, arxiv:0611291 [hep-th].

- [17] P. Mayr and S. Stieberger, Nucl. Phys. B412, 502 (1994).
- [18] K. Sfetsos and A. A. Tseytlin, Phys. Rev. D49, 2933 (1994).
- [19] B. Kleihaus, J. Kunz, K. Myklevoll, Phys. Lett. B605, 151 (2005).
- [20] C. Csaki, J. Erlich, T. J. Hollowood, Y. Shirman, Nucl. Phys. B581, 309 (2000).
- [21] M. Gremm, Phys. Lett. B478, 434 (2000).
- [22] C. Csaki, J. Erlich, T. J. Hollowood, Y. Shirman, Phys.Rev.Lett. 84, 5932-5935 (2000).